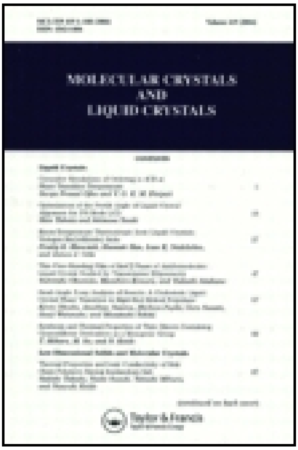

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Effect of External Field and Surfaces on Director Correlation Function in Nematic Liquid Crystals

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The correlation function of director fluctuations in a slab of nematic liquid crystal is calculated taking into account a surface energy and external electrical or magnetic field, oriented a liquid crystal in the same direction as a surface. The calculations have been carry out by the method of path integral with the separation bulk and surface degrees of freedom. The planar and homeotropic geometries are considered for positive and negative anisotropy of permittivity or magnetic susceptibility. The detailed results are presented for homeotropic orientation. The obtained results are applied for analyses of possibilities of the light scattering experiments in considered systems. In particular, is shown that there is a possibility to measure an anchoring energy from dependence of light scattering intensity on the value of applied magnetic or electric field.

Keywords: Nematic; anchoring; director fluctuations; external field

1 INTRODUCTION

Among the optical properties of the nematic liquid crystals (NLC) the light scattering phenomena is of the constant interest. While studying the light scattering in NLC one should know the correlation function $\widehat{G}$ of the director fluctuations. The feature of these fluctuations is their long-range character. The intensity of the single light scattering I in the infinite medium was studied by de Gennes [1]. In the absence of an external field the scattering intensity depends on the wave vector q as $I \sim q^{-2}$ and so it tends to infinity for $q \to 0$.

However, in the most of applications the thickness of the NLC cell is very small and therefore the accounting of the interaction between NLC molecules and boundary surfaces is required. The action of the boundary surfaces is similar to the action of the external field — they orient NLC and suppress the fluctuations.

In the present paper threshold phenomena when the external field and the surface tend to orient NLC in the different directions, e.g. Frederiks effect, are beyond our interests.

To calculate the correlation function of the director fluctuations in the finite NLC cell the new mathematical approaches should be used. They differs significantly from those used in [1] based on three dimensional Fourier transforms. In [2, 3, 4] the similar problem with the rigid boundary conditions was solved using the expansion of the correlation function over the eigenfunctions of the system and the obtained result was presented as the infinite series. The correlation function $\widehat{G}$ was obtained in the closed form in [5] for the finite anchoring energy using the properties of the self-adjoined operators. However the physical sense and limitations of the suggested method are unclear. More clear method of calculation the correlation function in the finite cells was used in [6] for NLC and in [7] for smectics. In these papers the separating of the bulk and surface degrees of freedom in the order parameter was used.

In the present work the method of path integrals and separation of bulk and surface degrees of freedom are used for obtaining the general expression for correlation function of the director fluctuations in the finite cells accounting the influence of the external fields and the effect of anchoring surfaces. Unlike the most of similar investigations we considered both positive and negative anisotropy of the susceptibility of the system to the external field. Also both planar and homeotropic geometries are considered. The correlation function for homeotropic orientation is analysed in detail.

2 PRESENTATION OF THE CORRELATION FUNCTION IN THE PATH INTEGRALS FORM

The free energy F_{tot} of NLC in the external field (in what follows we will consider a magnetic field **H**) could be presented as the sum of the bulk energy F_{bulk} and surface energy F_{sf}. Here

$$F_{bulk} = \frac{1}{2}\int d^3\mathbf{r}\Big[K_{11}(\operatorname{div}\mathbf{n})^2 + K_{22}(\mathbf{n}\operatorname{curl}\mathbf{n})^2 + K_{33}(\mathbf{n}\times\operatorname{curl}\mathbf{n})^2 - \chi_a\,(\mathbf{nH})^2\Big], \quad (1)$$

where K_{ii} are Frank modules, **n** is vector of the director, χ_a is the anisotropy of the magnetic susceptibility. The surface energy is determined by coupling the nematic molecules with bounded surfaces. In the most cases the anchoring energy is presented in the Rapini potential form [8]. Here we suppose that NLC is confined in a plane-parallel layer of L thickness. The system will be considered in Cartesian frame with the origin in the center of the cell and z axis directed normal to the layer, $-L/2 \le z \le L/2$, $\mathbf{n}_\perp = (n_x, n_y)$. For homeotropic orientation

$$F_{sf} = -\frac{1}{2}\int d^2\mathbf{r}_\perp\left[W_1 n_\perp^2(\mathbf{r}_\perp, -L/2) + W_2 n_\perp^2(\mathbf{r}_\perp, L/2)\right]. \quad (2)$$

For the planar orientation

$$F_{sf} = \frac{1}{2}\int d^2\mathbf{r}_\perp\big[W_{x1}n_x^2(\mathbf{r}_\perp, -L/2) + W_{x2}n_x^2(\mathbf{r}_\perp, L/2) + W_{y1}n_y^2(\mathbf{r}_\perp, -L/2) + W_{y2}n_y^2(\mathbf{r}_\perp, L/2)\big]. \quad (3)$$

Eq.(3) is written for Cartesian coordinate frame with y axis directed along the weak anchoring direction, $\mathbf{e}_y \parallel \mathbf{n}_0$, $\mathbf{n}_0$ is the equilibrium vector of the director. The coefficients W_i, W_{xi}, W_{yi} (i=1,2) are the densities of the anchoring energy.

As the system is assumed to be infinite in the x–y plain it is convenient to perform the two-dimensional Fourier transform over these coordinates $\mathbf{n}(x, y, z) \to \mathbf{n}(\mathbf{q}_\perp, z)$.

We calculate the correlation function in the $\mathbf{q}_\perp, z$ representation:

$$G_{\alpha\beta}(\mathbf{q}_\perp; z, z_1) = \langle \delta n_\alpha(\mathbf{q}_\perp, z)\, \delta n^*_\beta(\mathbf{q}_\perp, z_1)\rangle\,, \tag{4}$$

where $\delta\mathbf{n} = \mathbf{n} - \mathbf{n}_0$ and brackets mean the statistical averaging. In the Gaussian approximation the director modes with different $\mathbf{q}_\perp$ are statistically independent and we can write

$$G_{\alpha\beta}(\mathbf{q}_\perp, z, z_1) = \int \cdots \int \delta n_\alpha(\mathbf{q}_\perp, z)\delta n^*_\beta(\mathbf{q}_\perp, z_1)\rho(\delta\mathbf{n})\, \mathcal{D}\delta\mathbf{n}\,. \tag{5}$$

where

$$\rho(\delta\mathbf{n}) = \frac{1}{\mathbb{Z}} \exp\left[-\frac{\Phi(\delta\mathbf{n}, \mathbf{q}_\perp)}{k_B T}\right],$$
$$\mathbb{Z} = \int \cdots \int \exp\left[-\frac{\Phi(\delta\mathbf{n}, \mathbf{q}_\perp)}{k_B T}\right] \mathcal{D}\delta\mathbf{n}\,, \tag{6}$$

the potential $\Phi(\delta\mathbf{n}, \mathbf{q}_\perp)$ is determined by the equality

$$F_{tot} = \int \Phi(\delta\mathbf{n}, \mathbf{q}_\perp) \frac{d^2\mathbf{q}_\perp}{(2\pi)^2}\,.$$

Symbol $\int \cdots \int \mathcal{D}\delta\mathbf{n}$ means path integral over all possible system configurations of $\delta\mathbf{n}(\mathbf{q}_\perp, z)$ fluctuations with fixed value of $\mathbf{q}_\perp$, in particular it means integration over all possible values of $\delta\mathbf{n}(\mathbf{q}_\perp, z) \equiv \delta\mathbf{n}(z)$ on the surfaces $z = \pm L/2$ and in the volume $-L/2 \le z \le L/2$.

If we denote

$$\delta\mathbf{n}(L/2) = \mathbf{u}, \;\; \delta\mathbf{n}(-L/2) = \mathbf{v}, \tag{7}$$

Eq.(5) can be presented in the form

$$G_{\alpha\beta}(\mathbf{q}_\perp; z, z_1) = \frac{1}{\mathbb{Z}} \int \exp\left[-2\frac{\Phi_{sf}(\mathbf{u}, \mathbf{v})}{k_B T}\right] d\mathbf{u} d\mathbf{v}$$
$$\times \underbrace{\int \cdots \int}_{\delta\mathbf{n}(-L/2)=\mathbf{u},\, \delta\mathbf{n}(L/2)=\mathbf{v}} \exp\left(-\frac{2\Phi_{bulk}}{k_B T}\right) \delta n_\alpha(z)\delta n^*_\beta(z_1)\, \mathcal{D}\delta\mathbf{n}\,. \tag{8}$$

This presentation is convenient because the internal integral corresponds to the problem with the fixed values of $\delta\mathbf{n}$ on the boundaries.

Quite simple method of calculation of such an integral was proposed by Feynman [9]. If we present $\delta\mathbf{n}$ in the form

$$\delta\mathbf{n}(z) = \delta\mathbf{n}^{(0)}(z) + \boldsymbol{\eta}(z) \tag{9}$$

where $\delta\mathbf{n}^{(0)}(z)$ corresponds to the minimum of the free energy with the conditions (7) then $\boldsymbol{\eta}(\pm L/2) = 0$. Function $\delta\mathbf{n}^{(0)}(z)$ can be found by solving the corresponding Euler equation with the boundary conditions (7). Then from Eqs. (4), (9) we obtain

$$G_{\alpha\beta} = \langle \delta n_\alpha^{(0)} \delta n_\beta^{(0)} \rangle_{\mathbf{u},\mathbf{v}} + \langle \eta_\alpha \eta_\beta \rangle_\eta \,, \tag{10}$$

where $\langle \ldots \rangle_{\mathbf{u},\mathbf{v}}$ means the averaging over $\mathbf{u}, \mathbf{v}$ parameters with weight function $\exp(-\Phi^{(0)}/k_B T)$ corresponding to the $\delta\mathbf{n}^{(0)}$ configuration with the boundary conditions (7), $\langle \ldots \rangle_\eta$ means averaging over all possible configurations $\boldsymbol{\eta}$ with the weight function $\exp(-\Phi(\boldsymbol{\eta})/k_B T)$ with zeroth boundary conditions $\boldsymbol{\eta}(\pm L/2) = 0$. The first contribution to (10) corresponds to the finite-dimensional integral and the second to the path integral. The simplest way to calculate the second term is to use well known equality[10]

$$\langle \eta_\alpha(z) \eta_\beta^*(z_1) \rangle_\eta = -k_B T \, \frac{\delta}{\delta \epsilon_\beta(z_1)} \, \delta n_\alpha^{(0)}(z, \boldsymbol{\epsilon}) \Big|_{\boldsymbol{\epsilon}=0} \,. \tag{11}$$

where $\boldsymbol{\epsilon}$ is the fictitious external field ("source") which should be added to the right hand side of the Euler equation.

3 GENERAL EXPRESSION FOR CORRELATION FUNCTION IN NEMATIC SLAB

We consider the Euler equations for $\delta\mathbf{n}$ corresponding to the equilibrium condition $\delta F_{bulk} = 0$ for homeotropic and planar geometries for $\chi_a > 0$ and $\chi_a < 0$. In each case the suitable coordinate frame is chosen. Note, vector $\delta\mathbf{n}$ contains two fluctuating modes, δn_1 and δn_2. The determination of these modes will be given for every geometry.

1. *Homeotropic orientation,* $\chi_a > 0$. In this case the coordinate frame is chosen in such a manner that x axes is directed along $\mathbf{q}$, i. e. $q_y = 0$, $\mathbf{H} \parallel \mathbf{n}_0 \parallel \mathbf{e}_z$, and director fluctuation $\delta\mathbf{n} = (\delta n_1, \delta n_2, 0)$. Euler equation has the form

$$\left[\begin{pmatrix} K_{33} & 0 \\ 0 & K_{33} \end{pmatrix} \frac{\partial^2}{\partial z^2} - \begin{pmatrix} K_{11}q_\perp^2 + \chi_a H^2 & 0 \\ 0 & K_{22}q_\perp^2 + \chi_a H^2 \end{pmatrix}\right] \begin{pmatrix} \delta n_1 \\ \delta n_2 \end{pmatrix} = \epsilon(z). \quad (12)$$

2. *Homeotropic orientation,* $\chi_a < 0$. Here $\mathbf{n} \parallel \mathbf{e}_z$, $\mathbf{e}_x \parallel \mathbf{H}$, $\delta\mathbf{n} = (\delta n_1, \delta n_2, 0)$, and $\mathbf{q} = (q_1, q_2, 0)$. Euler equation is

$$\left[\begin{pmatrix} K_{33} & 0 \\ 0 & K_{33} \end{pmatrix} \frac{\partial^2}{\partial z^2} - \begin{pmatrix} K_{11}q_1^2 + K_{22}q_2^2 - \chi_a H^2 & (K_{11} - K_{22})q_1 q_2 \\ (K_{11} - K_{22})q_1 q_2 & K_{11}q_2^2 + K_{22}q_1^2 \end{pmatrix}\right] \begin{pmatrix} \delta n_1 \\ \delta n_2 \end{pmatrix} = \epsilon(z). \quad (13)$$

3. *Planar orientation,* $\chi_a > 0$. In this case $\mathbf{n}_0 \parallel \mathbf{H} \parallel \mathbf{e}_y$, $\mathbf{q} = (q_1, q_2, 0)$, $\delta\mathbf{n} = (\delta n_1, 0, \delta n_2)$. Euler equation has the form

$$\left[\begin{pmatrix} K_{11} & 0 \\ 0 & K_{22} \end{pmatrix} \frac{\partial^2}{\partial z^2} + iq_1 \begin{pmatrix} 0 & K_{11} - K_{22} \\ K_{11} - K_{22} & 0 \end{pmatrix} \frac{\partial}{\partial z} - \begin{pmatrix} K_{11}q_1^2 + K_{33}q_2^2 + \chi_a H^2 & 0 \\ 0 & K_{22}q_1^2 + K_{33}q_2^2 + \chi_a H^2 \end{pmatrix}\right] \times \begin{pmatrix} \delta n_1 \\ \delta n_2 \end{pmatrix} = \epsilon(z). \quad (14)$$

4. *Planar orientation,* $\chi_a < 0$. In this case $\mathbf{n}_0 \parallel \mathbf{e}_y$, $\mathbf{H} \perp \mathbf{e}_y$, $\mathbf{H} = H(\cos\alpha, 0, \sin\alpha)$, i.e. $\mathbf{H} \perp \mathbf{n}_0$, $\mathbf{q} = (q_1, q_2, 0)$, $\delta\mathbf{n} = (\delta n_1, 0, \delta n_2)$. Euler equation has the form

$$\left[\begin{pmatrix} K_{11} & 0 \\ 0 & K_{22} \end{pmatrix} \frac{\partial^2}{\partial z^2} + iq_1 \begin{pmatrix} 0 & K_{11} - K_{22} \\ K_{11} - K_{22} & 0 \end{pmatrix} \frac{\partial}{\partial z} - \begin{pmatrix} K_{11}q_1^2 + K_{33}q_2^2 - \chi_a H^2 \cos^2\alpha & -\chi_a H^2 \sin\alpha\cos\alpha \\ -\chi_a H^2 \sin\alpha\cos\alpha & K_{22}q_1^2 + K_{33}q_2^2 - \chi_a H^2 \sin^2\alpha \end{pmatrix}\right] \times \begin{pmatrix} \delta n_1 \\ \delta n_2 \end{pmatrix} = \epsilon(z). \quad (15)$$

Each of Eqs. (12)–(15) obeys the boundary conditions (7).

Equations (12) - (15) are the ordinary differential equations with the constant coefficients. The solution of each equation with the boundary conditions (7) could be written as

$$\delta\mathbf{n}^{(0)}(z) = \widehat{\Phi}(z)\widehat{M}\begin{pmatrix}\mathbf{v}\\ \mathbf{u}\end{pmatrix} + \int\limits_{l_1}^{l_2}\Big[\hat{k}(z,z') - \widehat{\Phi}(z)\widehat{M}\begin{pmatrix}\hat{1}\\ \hat{0}\end{pmatrix}\hat{k}(l_2,z')\Big]\boldsymbol{\epsilon}(z')dz' . \quad (16)$$

where $\hat{1}$ and $\hat{0}$ are unit and zeroth 2×2–matrices, $\widehat{\Phi}(z)$ is the 2×4–matrix of basic solutions of the corresponding homogeneous equation systems

$$\widehat{\Phi}(z) = (\boldsymbol{\phi}_1(z); \boldsymbol{\phi}_2(z); \boldsymbol{\phi}_3(z); \boldsymbol{\phi}_4(z)) , \quad (17)$$

where $\boldsymbol{\phi}_i(z)$ is the complete set of independent solutions of the homogeneous problem (without boundary conditions)

$$\boldsymbol{\phi}_i(z) = e^{\lambda_i z}\mathbf{e}_i \quad (18)$$

where λ_i are the eigenvalues and $\mathbf{e}_i$ are the eigenvectors of the problem,

$$\widehat{M} = \begin{pmatrix}\widehat{\Phi}(l_2)\\ \widehat{\Phi}(l_1)\end{pmatrix}^{-1}, \quad \hat{k}(z,z') = \theta(z-z')\widehat{\Phi}(z)\widehat{P}(z'),$$

$$\widehat{P}(z) = \begin{pmatrix}\hat{a}\widehat{\Phi}'(z)\\ \widehat{\Phi}(z)\end{pmatrix}^{-1}\begin{pmatrix}\hat{1}\\ \hat{0}\end{pmatrix} \quad (19)$$

where $\hat{a}$ is the diagonal matrix coefficient at $\partial^2/\partial z^2$ in Euler Eqs.(12), (13) and. Eqs.(14), (15).

The obtained results give the possibility to calculate the correlation functions of director fluctuations.

From Eqs.(16), (11) we can find the second term in the Eq.(10)

$$\langle\eta_\alpha(z)\eta^*_\beta(z_1)\rangle_\eta = k_BT\left[\widehat{\Phi}(z)\widehat{M}\begin{pmatrix}\hat{1}\\ \hat{0}\end{pmatrix}\hat{k}(l_2,z_1) - \hat{k}(z,z_1)\right] . \quad (20)$$

Assuming $\epsilon = 0$ and averaging over **u** and **v** which reduces to the calculation of the finite-dimension Gaussian integral we obtain for the first term in (10)

$$\langle \delta n^{(0)}\alpha(z)\delta n_{\beta}^{(0)*}(z_1)\rangle_{\mathbf{u},\mathbf{v}} = k_B T\left[\widehat{\Phi}(z)\widehat{M}\widehat{F}_1^{-1}\widehat{M}^{+}\widehat{\Phi}^{+}(z_1)\right]_{\alpha\beta}. \quad (21)$$

Here

$$\widehat{F}_1 = \left[\begin{pmatrix}\hat{1}\\ \hat{0}\end{pmatrix}\hat{a}\widehat{\Phi}'(l_2) - \begin{pmatrix}\hat{0}\\ \hat{1}\end{pmatrix}\hat{a}\widehat{\Phi}'(l_1)\right]\widehat{M} + \begin{pmatrix}\hat{b}+\widehat{w}_2 & \hat{0}\\ \hat{0} & -\hat{b}+\widehat{w}_1\end{pmatrix}. \quad (22)$$

here superscript "+" means Hermitian conjugation. Here for homeotropic orientation $\hat{b} = \hat{0}$, $\widehat{w}_i = W_i\,\hat{1}$ and for planar orientation

$$\hat{b} = iq_1\begin{pmatrix}0 & -K_{22}\\ K_{11} & 0\end{pmatrix}, \quad \widehat{w}_i = \begin{pmatrix}W_{xi} & 0\\ 0 & W_{zi}\end{pmatrix}. \quad (23)$$

As the result we obtain the ultimate expression for the correlation function of the director fluctuations in the finite cell:

$$\widehat{G}(z,z_1) = k_B T\Big[\widehat{\Phi}(z)\widehat{M}\widehat{F}_1^{-1}\widehat{M}^{+}\widehat{\Phi}^{+}(z_1) - \hat{k}(z,z_1) + \widehat{\Phi}(z)\widehat{M}\begin{pmatrix}\hat{1}\\ \hat{0}\end{pmatrix}\hat{k}(l_2,z_1)\Big]. \quad (24)$$

Equation (24) could be applied both in the case of homeotropic and planar geometries, but in the case of planar geometry the answer is more cumbersome. So to illustrate the using of the method we present the case of homeotropic geometry for $\chi_a > 0$ and $\chi_a < 0$.

4 EXPLICIT EXPRESSION FOR $\widehat{G}$ IN HOMEOTROPIC GEOMETRY

In the case of $\chi_a > 0$ for eigenvalues λ_i and eigenvectors $\mathbf{e}_i$ from Eq.(12) we obtain:

$$\lambda_i = -\lambda_{i+2} = \sqrt{\left(K_{ii}q_\perp^2 + \chi_a H^2\right)/K_{33}}, \quad i = 1,2;$$
$$\mathbf{e}_1 = \mathbf{e}_3 = \begin{pmatrix}0\\1\end{pmatrix}, \quad \mathbf{e}_2 = \mathbf{e}_4 = \begin{pmatrix}1\\0\end{pmatrix}. \quad (25)$$

For $\chi_a < 0$ from Eq.(13) we get:

$$\lambda_i = -\lambda_{i+2} = \sqrt{\left[(K_{11}+K_{22})q_\perp^2 - \chi_a H^2 + (-1)^{i+1}Q\right]/2K_{33}}\,,$$
$$Q = \sqrt{q_\perp^4(K_{11}-K_{22})^2 - 2\chi_a H^2(K_{11}-K_{22})(q_1^2-q_2^2) + \chi_a^2 H^4}\,, \quad (26)$$
$$\mathbf{e}_i = \mathbf{e}_{i+2} = \begin{pmatrix} (K_{11}-K_{22})q_1q_2 \\ K_{33}\lambda_i^2 - K_{11}q_1^2 - K_{22}q_2^2 - \chi_a H^2 \end{pmatrix}, \quad i = 1,2.$$

Notice that vectors $\mathbf{e}_i$ may not be unit ones.

Substituting Eqs.(25), (27) into the Eqs.(16)-(24) we obtain the results in the corresponding coordinate frame.

Let's introduce the notations

$$\begin{aligned}\mathcal{E}_l(z,z') = \big[&\left(W^2+\lambda_l^2K_{33}^2\right)\cosh\left(\lambda_l\left(L-|z-z'|\right)\right) + \\ &+ 2W\lambda_lK_{33}\sinh\left(\lambda_l\left(L-|z-z'|\right)\right) \\ &- \left(W^2-\lambda_l^2K_{33}^2\right)\cosh\left(\lambda_l(z+z')\right)\big] \\ \times &\left[\left(W^2+\lambda_l^2K_{33}^2\right)\lambda_l\sinh\left(\lambda_lL\right) + 2W\lambda_l^2K_{33}\cosh\left(\lambda_lL\right)\right]^{-1}\end{aligned} \quad (27)$$

In the case of $\chi_a > 0$, $G_{12} = G_{21} = 0$ and

$$G_{ll}(z,z') = \frac{k_BT\mathcal{E}_l(z,z')}{2K_{33}}, \quad l = 1,2. \quad (28)$$

In the case of $\chi_a < 0$ calculations are more complicated in comparison to $\chi_a > 0$ case, but the final results are as compact as Eq.(28)

$$\begin{aligned}G_{ll}\left(\mathbf{q}_\perp,z,z'\right) &= \frac{k_BT}{2Q}\sum_{i=1,2}(-1)^i\left(A_l-\lambda_i^2\right)\mathcal{E}_i(z,z'),\ l=1,2\,,\\ G_{12}(\mathbf{q}_\perp,z,z') &= \frac{k_BTB}{2Q}\sum_{i=1,2}(-1)^{i+1}\mathcal{E}_i(z,z')\,, \quad G_{21}=G_{12}\,,\end{aligned} \quad (29)$$

where

$$A_1 = \left(K_{11}q_2^2+K_{22}q_1^2\right)/K_{33}\,,\ B = q_1q_2\left(K_{11}-K_{22}\right)/K_{33}\,,$$
$$A_2 = \left(K_{11}q_1^2+K_{22}q_2^2-\chi_aH^2\right)/K_{33}\,. \quad (30)$$

5 DISCUSSION

Let's analyse the obtained results. First of all we consider the case of infinite cell (i.e. $L \to \infty$), when the action of surface can be vanished. In the limit $L \to \infty$ from Eqs.(27) one can see that all terms of correlation matrices (28), (29) depend on z and z' as $|z - z'|$ only, since

$$\lim_{L\to\infty} \mathcal{E}_l(z, z') = \mathcal{E}_l(z - z') = \frac{e^{-\lambda_l|z-z'|}}{\lambda_l}. \tag{31}$$

To compare with known results it is suitable to make a Fourier - transformation over $z - z'$. We take into account that

$$\mathcal{E}_l(q_\parallel) = \int_{-\infty}^{\infty} \mathcal{E}_l(z) e^{-iq_\parallel z}\, dz = \frac{2}{\lambda_l^2 + q_\parallel^2}. \tag{32}$$

Then for $\chi_a > 0$ we have from Eq.(28)

$$G_{ll}(\mathbf{q}) = \frac{k_B T}{K_{ll}q_\perp^2 + K_{33}q_\parallel^2 + \chi_a H^2}, \tag{33}$$

$(l = 1, 2)$, $G_{12}(\mathbf{q}) = 0$, $\mathbf{q} = (\mathbf{q}_\perp, q_\parallel)$. This expressions coincide with well-known de Gennes results [1]. For $q \to 0$ and $H \neq 0$ the values G_{11} and G_{22} are finite. It means that the fluctuations of both modes, δn_1 and δn_2, are suppressed by the external field.

For $\chi_a < 0$ Eqs.(29), (32) give

$$\begin{aligned} G_{11}(\mathbf{q}) &= \left(K_{11}q_2^2 + K_{22}q_1^2 + K_{33}q_\parallel^2\right) P(\mathbf{q}; \mathbf{H}), \\ G_{12}(\mathbf{q}) &= \left(K_{22} - K_{11}\right) q_1 q_2 P(\mathbf{q}; \mathbf{H}), \\ G_{22}(\mathbf{q}) &= \left(K_{11}q_1^2 + K_{22}q_2^2 + K_{33}q_\parallel^2 - \chi_a H^2\right) P(\mathbf{q}; \mathbf{H}), \end{aligned} \tag{34}$$

where

$$P(\mathbf{q}; \mathbf{H}) = k_B T \left[\left(K_{11}q_\perp^2 + K_{33}q_\parallel^2\right)\left(K_{22}q_\perp^2 + K_{33}q_\parallel^2\right) - \chi_a H^2 \left(K_{11}q_2^2 + K_{22}q_1^2 + K_{33}q_\parallel^2\right)\right]^{-1}. \tag{35}$$

In the case of strong fields when $|\chi_a| H^2 \gg K_{ll}q^2$, $(l = 1\text{–}3)$ we get from Eqs.(34), (35):

$$\begin{aligned} G_{11}(\mathbf{q}) &= \frac{k_B T}{|\chi_a| H^2}, \\ G_{12}(\mathbf{q}) &= \frac{k_B T}{|\chi_a| H^2} \cdot \frac{(K_{22} - K_{11}) q_1 q_2}{K_{11} q_2^2 + K_{22} q_1^2 + K_{33} q_{\|}^2}, \\ G_{22}(\mathbf{q}) &= \frac{k_B T}{K_{11} q_2^2 + K_{22} q_1^2 + K_{33} q_{\|}^2}. \end{aligned} \tag{36}$$

It is seen from Eqs.(36) that in limit $q \to 0$ the behaviour of the correlation matrix elements distinct significantly. Elements G_{11} and G_{12} remain finite when $q \to 0$, and element G_{22} increases infinitely. It means that in the case of $\chi_a < 0$ the external field suppresses one fluctuation mode only, namely δn_1 mode. The mode δn_2 remains Goldstone even in the presence of the external field.

Let's analyse the case of finite L and $H \to 0$. For $\chi_a < 0$ an there is an arbitrariness in choosing of x axis direction. If we direct this axis along $\mathbf{q}_\perp$ as well as for $\chi_a > 0$ (here it is necessary to remove indeterminacy at $B = 0$), we can see that expressions (29) and (28) coincide. They are similar to Eqs.(28), with $\lambda_1 = q_\perp (K_{11}/K_{33})^{1/2}$, $\lambda_2 = q_\perp (K_{22}/K_{33})^{1/2}$. These expressions are identical to results of Ref.[5].

It is of interest to compare the correlation function $\widehat{G}^L = \widehat{G}(H = 0, \mathbf{q}_\perp, z, z')$ in bounded cell with the corresponding inverse Fourier transform $\widehat{G}^H = G(L = \infty, \mathbf{q}_\perp, z, z')$ correlation function in the unbounded specimen in the external field. The director fluctuations in the unbounded specimen for $H = 0$ are of the Goldstone type $G_{ll}(\mathbf{q}) \sim q^{-2} \to \infty$ for $q \to 0$. When $H \neq 0$, from Eq.(33)

$$G_{ll}^H(\mathbf{q} = 0) = \int_{-\infty}^{\infty} dz' \, G_{ll}^H(\mathbf{q}_\perp = 0, z - z') = \frac{k_B T}{K_{33}} \frac{1}{\xi_H^2}, \tag{37}$$

where $\xi_H = H(\chi_a/K_{33})^{1/2}$ is inverse magnetic coherence length.

After removal of the indeterminacy at $\mathbf{q}_\perp \to 0$ in the correlation function (28) for a bounded cell the analogous integral has the following form

$$F(W,L,z) = \int_{-L/2}^{L/2} dz'\, G^L_{ll}(\mathbf{q}_\perp = 0, z, z')$$

$$= \frac{k_B T}{K_{33}} \frac{1}{2\xi_W} \left[L + \xi_W \left(\frac{L^2}{4} - z^2 \right) \right], \quad (38)$$

where $\xi_W = W/K_{33}$ is the inverse length which characterisese the anchoring of nematic crystal to the substrate.

For $L \to \infty$ and $\xi_H \to 0$, both equations, (37) and (38), diverge quadratically. It reflects the Goldstone nature of director fluctuations for $L = \infty$ and $H = 0$. However, it is significant that, for a finite L, the quantity defined by Eq.(38) diverges linearly if $\xi_W \to 0$. Thus not only in bounded medium but also in thin nematic liquid crystal films the director fluctuations are singular in the absence of anchoring to surface.

In the case of finite L and finite H it is possible to make the following conclusions. For $\chi_a > 0$ the surface and external field suppress the fluctuations of the both δn_1 and δn_2 modes. For $\chi_a < 0$ both δn_1 and δn_2 modes depend on the energy of interaction with surface W in a similar way. On the other hand, the field acts upon the δn_1 mode more significant than upon the δn_2 mode.

To illustrate a simultaneous influence of the external field and the surface anchoring in Eq.(28) we calculated the function

$$F(W,L,H,z) = \int_{-L/2}^{L/2} dz'\, G_{ll}(\mathbf{q}_\perp = 0, z, z') =$$

$$= \frac{k_B T}{K_{33}\xi_H^2} \left[1 - \cosh(\xi_H z) \frac{2\xi_W \exp(-\xi_H L/2)}{\xi_W + \xi_H + (\xi_W - \xi_H)\exp(-\xi_H L)} \right]. \quad (39)$$

(function $F(W,L,z)$ in the Eq.(38) and function $F(W,L,H,z)$ in the Eq.(39) obey the following relation $F(W,L,H \to 0,z) = F(W,L,z)$).

Figure 1 shows the dependence of this function on H and W for $z = 0$. It is seen that the dependence of F on the external field is very sensitive to W value. So for $W = 10^{-4}\,\text{erg/cm}^2$ the value of F increases by the factor of 12 while H varies from 500 Gs to 5000 Gs, whereas for $W = 10^{-5}\text{erg/cm}^2$ this function increases by

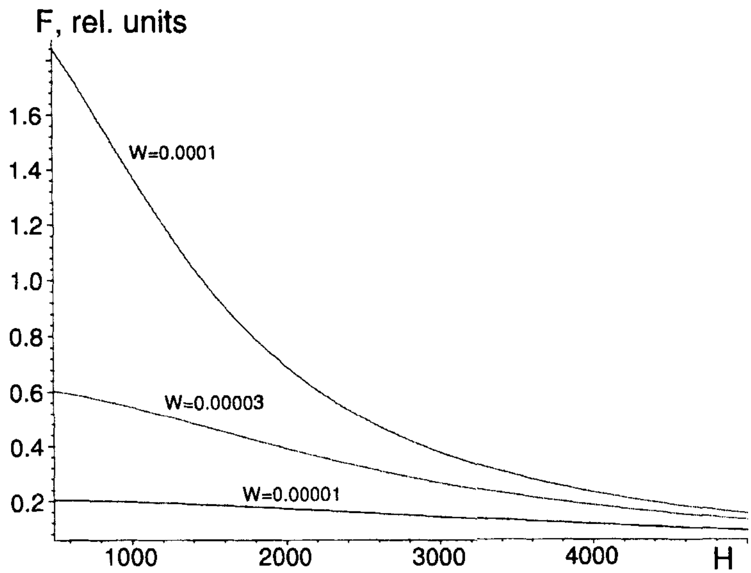

FIGURE 1

Dependence of $F(W, L, H, z = 0)$ function in Eq.(39) on the external field H for three values of the anchoring energy W. Typical parameters $K_{33} = 10^{-6}$ dynes, $\chi_a = 10^{-7}$, $L = 10^{-4}$ cm, $10^{-4} \leqslant W \leqslant 10^{-5}$ erg/cm^2, $500 \leqslant H \leqslant 5000$ Gs were used.

the factor of 2.2 only. Thus, the measurement of the correlation function for various fields can be used for determination of surface anchoring energy.

Experimentally, it can be fulfilled by measuring the distribution of the scattered light of different polarisations in external fields.

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